Zoom Class 1/5/2020

Asymptote Problems

Ques Find the asymptotes of the curve
$$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$$

Soln Given $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$
Since the highest power of x is x^3 and co-efficient of x^3 is 1, therefore there is no asymptote parallel to x-axis..

Now putting $x = 1$ and $y = m$ in 3rd degree terms we get $\phi_3(m) = 1 - 2m + m^2 = 0$

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$$\Rightarrow (m-1)^2 = 0 \Rightarrow m = 1$$

Again $\phi_2(m) = 1 - m$
and $\phi_3'(m) = 2(m-1)$

$$\therefore c = -\frac{\phi_2(m)}{\phi_3'(m)} = -\frac{(1-m)}{2(m-1)}$$
$$= \frac{(1-m)}{2(1-m)}$$

$\therefore$ For $m = 1$, $c = \frac{0}{0}$

$\therefore$ We take $\frac{c^2}{2!}\phi_3''(m) + c\,\phi_2'(m) + \phi_1(m) = 0$

$$\Rightarrow \frac{c^2}{2}(2) + c(-1) + 0 = 0$$

$$\Rightarrow c^2 - c = 0 \Rightarrow c(c-1) = 0$$

$\Rightarrow c = 0, 1$

Also since y^3 is not present the co-efficient of y^2 is $x = 0$

$\therefore$ The required co-efficients are

$$y = 1\cdot x + 0 \Rightarrow y = x$$
$$y = 1\cdot x + 1 \Rightarrow y = x + 1$$
$$x = 0$$

Ques: Find the asymptotes of the curve $(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ and the prove that the asymptotes for a triangle of area 'a^2'.

Soln Given $(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$

$\Rightarrow x^2y - y^3 - 2ay^2 + 5x - 7 = 0$

Putting $x = 1$ and $y = m$ in highest power we get

$$\phi_3(m) = m - m^3 = 0$$
$$\Rightarrow m(1 - m^2) = 0$$

$\Rightarrow m = 0, m = 1, -1$

$\phi_3'(m) = 1 - 3m^2$

$\phi_2(m) = -2am^2$

$$c = -\frac{\phi_2(m)}{\phi_3'(m)} = -\frac{(-2am^2)}{(1 - 3m^2)} = \frac{2am^2}{(1 - 3m^2)}$$

For $m=0$, $c = \dfrac{2a\cdot 0}{1-3\cdot 0} = 0$

For $m=1$, $c = \dfrac{2a\cdot 1}{1-3\cdot 1} = \dfrac{2a}{-2} = -a$

for $m=-1$, $c = \dfrac{2a(-1)^2}{1-3(-1)^2}$

$= \dfrac{2a}{-2} = -a$

$\therefore$ Required asymptotes are

$y = 0.x+0 \Rightarrow y = 0$ —— ①

$y = 1\cdot x - a \Rightarrow y = x-a$ —— ②

$y = -1\cdot x - a \Rightarrow y = -x-a$ —— ③

We use the above asymptotes to find vertices of the triangle.

Solving ① & ② we get $y=0$, $x=a$

Solving ① & ③ we get $y=0$, $x=-a$

Solving ② & ③ we get $y=-a$, $x=0$.

Thus co-ordinates are $(a,0), (-a,0), (0,-a)$

$\therefore$ Area of triangle is

$\frac{1}{2}[a(0+a) + (-a)(-a-0) + 0(0-0)]$

$= \frac{1}{2}[a^2+a^2] = \frac{2a^2}{2} = a^2$

Proved